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# STRAIN-RATE INTENSITY FACTOR IN COMPRESSION OF A LAYER OF A PLASTIC MATERIAL BETWEEN CYLINDRICAL SURFACES

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*A model problem for a rigid perfectly/plastic material is obtained. Based on this solution, it is possible to estimate the influence of the friction surface curvature and one of the types of additional rotational motion of the friction surface on the strain-rate intensity factor.*

**Key words:** *strain-rate intensity factor, singularity, friction, plasticity.*

The strain-rate intensity factor introduced in [1] as a coefficient at the principal singular term in the expansion of the equivalent strain rate into a series in the vicinity of the maximum friction surface is used to predict the evolution of material properties in a thin layer near surfaces with large friction stresses in metal-forming processes [2, 3]. Quantitative dependences between the strain-rate intensity factor and parameters characterizing material properties, however, have not been established. It seems of interest, therefore, to find the dependence of the strain-rate intensity factor on parameters of metal-forming processes, which can be used, in particular, to determine the above-mentioned quantitative dependences. One parameter that can be readily changed in an experiment is the curvature of the friction surface. In the present paper, we study the compression of a layer of a plastic material between two concentric cylinders with the maximum friction law being satisfied on the cylinder surfaces. This simple model problem allows the effect of the friction surface curvature on the strain-rate intensity factor to be estimated.

It was found that the strain-rate intensity factor is affected by additional rotation of the instrument, which is used in industrial processes for changing the energy and force parameters of the process [4–6]. The effect of one type of additional rotation of the friction surface on the strain-rate intensity factor is considered in the present paper within the framework of the model problem being solved.

For some processes, the strain-rate intensity factor was determined in [7–10], where the solutions were obtained on the basis of the double shear model [11], which is the generalization of the model of a rigid perfectly/plastic material. Note that the qualitative behavior of the solutions obtained on the basis of the double shear model coincides with the behavior of the solutions obtained on the basis of the model of a rigid perfectly/plastic material [12, 13].

Let us consider a plane flow of a layer of a plastic material compressed between two surfaces, which have the form of concentric circular cylinders and which obey the maximum friction law. We introduce a polar coordinate system $(r, \theta)$ whose origin coincides with the cylinder centers. The radius of the outer cylinder determined by the equation $r = R_2$ is assumed to be constant. The radius of the inner cylinder increases with a velocity $U_0$, and its current radius is described by the equation $r = R_1$ (Fig. 1). As the flow is symmetric about the axis $\theta = 0$, it is sufficient to construct the solution for $\theta \geqslant 0$. Let $\sigma_{rr}$, $\sigma_{\theta\theta}$, and $\sigma_{r\theta}$ be the components of the stress tensor in the polar coordinate system, and let $u_r$ and $u_\theta$ be the components of the velocity vector. The static boundary conditions have the following form:

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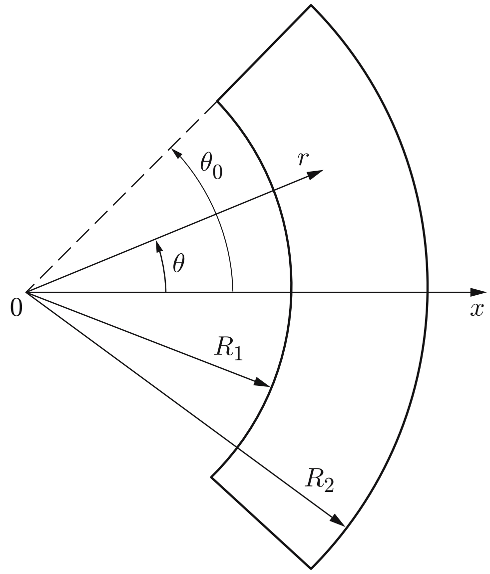


Fig. 1. Geometry of the problem.

— on the edge of the layer ($\theta = \theta_0$),

$$\sigma_{\theta\theta} = 0, \qquad \sigma_{r\theta} = 0;$$

— on the axis of symmetry ($\theta = 0$),

$$\sigma_{r\theta} = 0.$$

With allowance for the flow direction, the maximum friction law takes the form

$$\begin{aligned} \sigma_{r\theta} &= +k \quad \text{for} \quad r = R_1, \\ \sigma_{r\theta} &= -k \quad \text{for} \quad r = R_2 \end{aligned} \tag{1}$$

($k$ is the shear yield stress of the material). The kinematic boundary conditions are

$$u_r = 0 \quad \text{for} \quad r = R_2; \tag{2}$$

$$u_r = U_0 \quad \text{for} \quad r = R_1; \tag{3}$$

$$u_\theta = 0 \quad \text{for} \quad \theta = 0.$$

A comparison of the above-given formulation of the problem and the formulation of the Prandtl problem on compression of a layer between parallel plates [14] shows that the problem considered is the generalization of the Prandtl problem. Hence, an acceptable approximate solution can be obtained if the layer thickness is rather small: $(R_2 - R_1)/(R_2\theta_0) \ll 1$. In this case, however, the boundary conditions at $\theta = 0$ and $\theta = \theta_0$ cannot be satisfied with sufficient accuracy and should be replaced by the integral conditions

$$\int_{R_1}^{R_2} \sigma_{\theta\theta}\Big|_{\theta=\theta_0} dr = 0; \tag{4}$$

$$\int_{R_1}^{R_2} u_\theta\Big|_{\theta=0} dr = 0. \tag{5}$$

It follows from conditions (5) that the volume of the material extruded by the surface $r = R_1$ should be equal to the mass flow of the material through the surface $\theta = \theta_0$. In the exact solution, rigid zones are formed near the surfaces $\theta = 0$ and $\theta = \theta_0$.

The system of the static equations of a plane flow of a rigid perfectly/plastic material consists of the equilibrium equations and the yield condition, which has the form $(\sigma_{rr} - \sigma_{\theta\theta})^2 + 4\sigma_{r\theta}^2 = 4k^2$ in the polar coordinate system and is satisfied with the standard substitution [14]

$$\sigma_{rr} = \sigma + k\cos 2\psi, \qquad \sigma_{\theta\theta} = \sigma - k\cos 2\psi, \qquad \sigma_{r\theta} = k\sin 2\psi. \tag{6}$$

As we have $\sigma_{rr} \leqslant \sigma_{\theta\theta}$ in the problem considered, we obtain the following inequality from Eqs. (6):

$$\cos 2\psi \leqslant 0. \tag{7}$$

With allowance for inequality (7) and the last relation in Eqs. (6), the boundary conditions (1) acquire the form

$$\psi = \pi/4 \qquad \text{at} \quad r = R_1; \tag{8}$$

$$\psi = 3\pi/4 \qquad \text{at} \quad r = R_2. \tag{9}$$

It follows from conditions (8) and (9) that the quantity $\psi$ is independent of $\theta$. Then, the substitution of Eqs. (6) into the equilibrium equations in the polar coordinates yields

$$\frac{r}{k}\frac{\partial\sigma}{\partial r} - 2r\sin 2\psi\,\frac{d\psi}{dr} + 2\cos 2\psi = 0, \qquad \frac{1}{k}\frac{\partial\sigma}{\partial\theta} + 2r\cos 2\psi\,\frac{d\psi}{dr} + 2\sin 2\psi = 0. \tag{10}$$

System (10) is compatible under the condition

$$\sigma/k = A\theta + \sigma_0(r), \tag{11}$$

where the parameter $\sigma_0$ depends only on $r$; $A = \text{const}$. Substituting Eq. (11) into the second equation of system (10), we find

$$A + 2r\cos 2\psi\,\frac{d\psi}{dr} + 2\sin 2\psi = 0. \tag{12}$$

Integrating Eq. (12), we obtain

$$\sin 2\psi = C/r^2 - A/2, \qquad C = \text{const}. \tag{13}$$

It follows from the boundary conditions (8) and (9) that

$$A = 2(R_2^2 + R_1^2)/(R_2^2 - R_1^2), \qquad C = 2R_1^2R_2^2/(R_2^2 - R_1^2). \tag{14}$$

Using the formula $\partial\sigma/\partial r = (\partial\sigma/\partial\psi)(d\psi/dr)$, we replace the derivative $\partial\sigma/\partial r$ in the first equation of system (10). Using Eqs. (12) and (11) and eliminating the derivative $d\psi/dr$ and the parameter $\sigma$ from this equation, we obtain

$$\frac{d\sigma_0}{d\psi} = 2\,\frac{2 + A\sin 2\psi}{A + 2\sin 2\psi}. \tag{15}$$

The solution of Eq. (15) can be found in quadratures. For convenience, however, we write the solution in the form

$$\sigma_0 = 2\int_{\pi/4}^{\psi} \frac{2 + A\sin 2\chi}{A + 2\sin 2\chi}\,d\chi + C_1, \tag{16}$$

where $C_1$ is the constant of integration, which is found from condition (4). Substituting expressions (11) and (16) into Eqs. (6), we present the normal components of the stress tensor in the form

$$\begin{aligned} \frac{\sigma_{rr}}{k} &= A\theta + 2\int_{\pi/4}^{\psi} \frac{2 + A\sin 2\chi}{A + 2\sin 2\chi}\,d\chi + \cos 2\psi + C_1, \\ \frac{\sigma_{\theta\theta}}{k} &= A\theta + 2\int_{\pi/4}^{\psi} \frac{2 + A\sin 2\chi}{A + 2\sin 2\chi}\,d\chi - \cos 2\psi + C_1. \end{aligned} \tag{17}$$

Taking into account that $dr = (dr/d\psi)\,d\psi$ and the quantities $dr/d\psi$ and $r$ are expressed as functions of $\psi$ with the use of relations (12) and (13), respectively, we obtain the following relation from Eqs. (4) and (17):

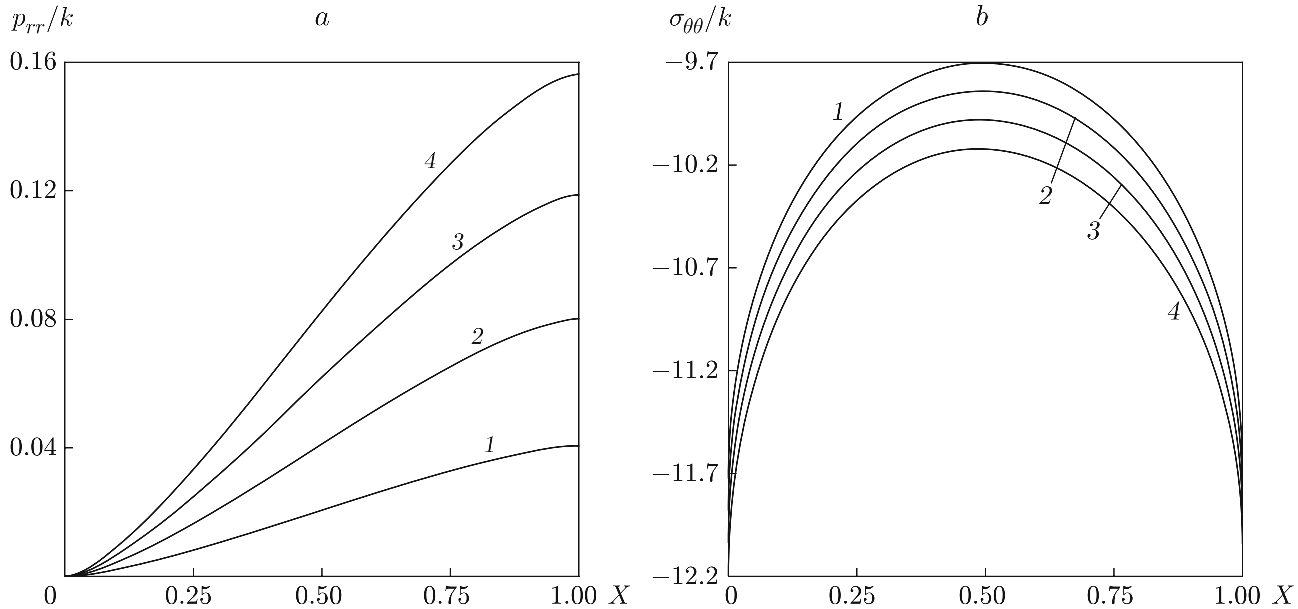


Fig. 2. Parameters $p_{rr}$ (a) and $\sigma_{\theta\theta}$ (b) as functions of the dimensionless coordinate $X$ for $\theta = \theta_0/2$, $R_1\theta_0/(R_2 - R_1) = 10$, and $\theta_0 = 15^\circ$ (1), $30^\circ$ (2), $45^\circ$ (3) and $60^\circ$ (4).

$$C_1 = \frac{2\sqrt{2}\sqrt{C}}{R_2 - R_1} \int\limits_{\pi/4}^{3\pi/4} \Big(2 \int\limits_{\pi/4}^{\psi} \frac{2 + A\sin 2\chi}{A + 2\sin 2\chi}\, d\chi - \cos 2\psi\Big) \frac{\cos 2\psi}{(A + 2\sin 2\psi)^{3/2}}\, d\psi - A\theta_0. \tag{18}$$

Using the formulas

$$p_1 = -\frac{1}{kR_1\theta_0} \int\limits_0^{\theta_0} \sigma_{rr}\Big|_{\psi=\pi/4} R_1\, d\theta, \qquad p_2 = -\frac{1}{kR_2\theta_0} \int\limits_0^{\theta_0} \sigma_{rr}\Big|_{\psi=3\pi/4} R_2\, d\theta,$$

we introduce the mean pressure per unit length on the instrument surfaces $r = R_1$ and $r = R_2$, respectively. Then, Eq. (17) yields

$$p_1 = -\Big(\frac{A\theta_0}{2} + C_1\Big), \qquad p_2 = -\Big(\frac{A\theta_0}{2} + C_2 + C_1\Big), \qquad C_2 = 2 \int\limits_{\pi/4}^{3\pi/4} \frac{2 + A\sin 2\chi}{A + 2\sin 2\chi}\, d\chi. \tag{19}$$

The resultant solution tends in the limit to the Prandtl solution [14]. Indeed, for $R_2 \to \infty$, $R_1 \to \infty$, and $\theta_0 \to 0$, the geometric scheme of the process corresponds to the Prandtl solution. We assume that $R_1\theta_0 \to l$ as $\theta_0 \to 0$, and $R_2 - R_1 = 2b$. Then, $R_2\theta_0 \to l$ as $\theta_0 \to 0$. Under these assumptions, it follows from Eq. (14) that $A \to \infty$ as $\theta_0 \to 0$, and Eq. (19) shows that $C_2 \to 0$ and $p_1 \to p_2$ as $\theta_0 \to 0$. In addition, we obtain from Eqs. (14) and (18) that $C_1 \to -A\theta_0 - \pi/2$, and Eqs. (15) and (19) yield the relation $p_1 \to p_2 \to l/(2b + \pi/2)$ for $\theta_0 \to 0$, which gives in the limit the mean pressure predicted by the Prandtl solution.

The components of the stress tensor as functions of $r$ and $\theta$ are determined from Eqs. (6), (13), (14), (17), and (18) in a parametric form. For a clearer illustration of the solution, we introduce a dimensionless coordinate $X = (r - R_1)/(R_2 - R_1)$, where $0 \leqslant X \leqslant 1$. As the stress $\sigma_{rr}$ in this interval changes insignificantly, as compared with its absolute value, we introduce a function $p_{rr} = \sigma_{rr} - \sigma_{rr}\Big|_{r=R_1}$. Figure 2 shows the parameters $p_{rr}$ and $\sigma_{\theta\theta}$ as functions of the coordinate $X$ for $\theta = \theta_0/2$, $R_1\theta_0/(R_2 - R_1) = 10$, and different angles $\theta_0$. (For $\theta_0 = 15$, 30, 45, and $60^\circ$, we have $(\sigma_{rr}/k)\Big|_{r=R_1} = -11.72$, $-11.88$, $-12.04$, and $-12.20$, respectively.) The dependence $\sigma_{r\theta}(X)$ is almost a straight line. Numerical calculations by formulas (19) show that the pressures $p_1$ and $p_2$ depend weakly on the parameter $\theta_0$. Thereby, we have $p_2 \leqslant p_1$, and the pressures become identical only as $\theta_0 \to 0$ (Prandtl solution).

The kinematic equations are determined by the associated flow rule and are reduced to the incompressibility equation and the equation expressing the condition of coaxiality of the stress and strain-rate tensors.

Let

$$u_r = U_0 U_r(r), \tag{20}$$

where $U_r(r)$ is an arbitrary function of $r$, which, by virtue of validity of Eqs. (2) and (3), should satisfy the conditions

$$U_r = 0 \qquad \text{at} \quad r = R_2; \tag{21}$$

$$U_r = 1 \qquad \text{at} \quad r = R_1. \tag{22}$$

Substituting relation (20) into the incompressibility equation and integrating the resultant expression, we obtain

$$u_\theta = -U_0\Big(r\,\frac{dU_r}{dr} + U_r\Big)\theta + U_0 U_\theta(r). \tag{23}$$

With allowance for Eqs. (20) and (23), the condition of coaxiality of the stress and strain-rate tensors implies that

$$\Big(\frac{U_r}{r} - \frac{dU_r}{dr} - r\,\frac{d^2U_r}{dr^2}\Big)\theta = \frac{U_\theta}{r} - \frac{dU_\theta}{dr} + 2\tan\,2\psi\,\frac{dU_r}{dr}. \tag{24}$$

Equation (24) has a solution if the following conditions are satisfied:

$$\frac{U_r}{r} - \frac{dU_r}{dr} - r\,\frac{d^2U_r}{dr^2} = 0, \qquad \frac{U_\theta}{r} - \frac{dU_\theta}{dr} + 2\tan\,2\psi\,\frac{dU_r}{dr} = 0. \tag{25}$$

The solution of the first equation of system (25) satisfying the boundary conditions (21) and (22) acquires the form

$$U_r = \frac{R_1(r^2 - R_2^2)}{r(R_1^2 - R_2^2)}. \tag{26}$$

Substituting solution (26) into the second equation of system (25), integrating it, and taking into account Eqs. (12)–(14), we obtain

$$U_\theta = \frac{\sqrt{2}\,R_2}{[R_2^2 + R_1^2 + (R_2^2 - R_1^2)\sin 2\psi]^{1/2}}$$

$$\times\Big(u_0 + \int\limits_{\pi/4}^{\psi} \frac{[R_2^2 + 3R_1^2 + (R_2^2 - R_1^2)\sin 2\gamma]\sin 2\gamma}{R_2^2 + R_1^2 + (R_2^2 - R_1^2)\sin 2\gamma}\,d\gamma\Big). \tag{27}$$

Here $u_0$ is the constant of integration whose value is found from condition (5). Passing in this condition to integration with respect to $\psi$ with the help of Eq. (12) and using expressions (13), (23), (26), and (27), we find

$$u_0 = 4R_1^2R_2^2 \int\limits_{\pi/4}^{3\pi/4} \frac{\cos 2\psi}{[(R_2^2 - R_1^2)\sin 2\psi + R_2^2 + R_1^2]^2}\Big(\int\limits_{\pi/4}^{\psi} \frac{[(R_2^2 - R_1^2)\sin 2\gamma + R_2^2 + 3R_1^2]\sin 2\gamma}{(R_2^2 - R_1^2)\sin 2\gamma + R_2^2 + R_1^2}\,d\gamma\Big)d\psi. \tag{28}$$

From Eqs. (13), (23), and (26)–(28), we obtain the dependences $u_r(r)$ and $u_\theta(\theta)$. The dependence $u_\theta(X)$ for $\theta = \theta_0/2$, $R_1\theta_0/(R_2 - R_1) = 10$, and different angles $\theta_0$ is plotted in Fig. 3.

It was demonstrated in [1] that the equivalent strain rate near the maximum friction surfaces obeys the law

$$\xi_{\rm eq} = Ds^{-1/2} + o(s^{-1/2}), \qquad s \to 0, \tag{29}$$

where $D$ is the strain-rate intensity factor and $s$ is the distance to the friction surface. In the case considered, the equivalent strain rate is expressed via the components of the strain-rate tensor $\xi_{rr}$, $\xi_{\theta\theta}$, and $\xi_{r\theta}$ in the polar coordinate system:

$$\xi_{\rm eq} = \sqrt{2/3}\,(\xi_{rr}^2 + \xi_{\theta\theta}^2 + 2\xi_{r\theta}^2)^{1/2}. \tag{30}$$

To determine the strain-rate intensity factor, we find the shear strain rate. It follows from Eqs. (20), (25), and (26) that

$$\xi_{r\theta} = \frac{U_0R_1(R_2^2 + r^2)}{(R_1^2 - R_2^2)r^2}\,\tan\,2\psi. \tag{31}$$

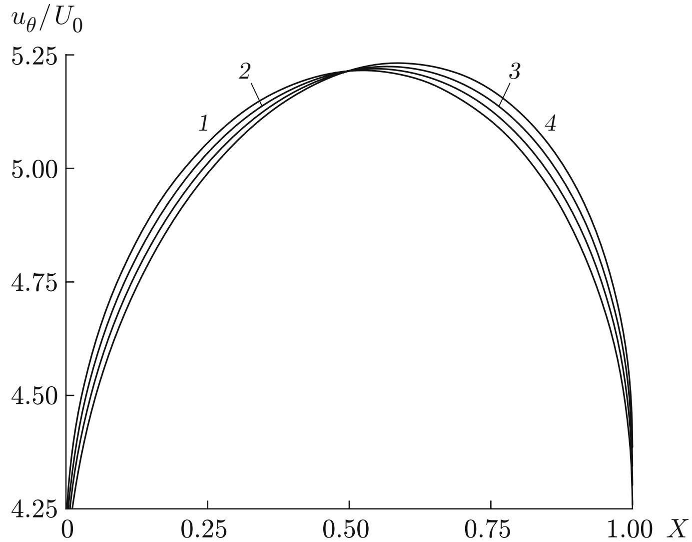


Fig. 3. Velocity component $u_\theta$ versus the dimensionless coordinate $X$ for $\theta = \theta_0/2$, $R_1\theta_0/(R_2-R_1) = 10$, and $\theta_0 = 15°$ (1), 30° (2), 45° (3), and 60° (4).

Expanding the quantity $\tan 2\psi$ into a series in the vicinity of the points $\psi = \pi/4$ and $\psi = 3\pi/4$ and taking into account Eqs. (8) and (9), we obtain the following relations from Eq. (31):

$$\xi_{r\theta} = \frac{U_0(R_2^2 + R_1^2)}{2R_1(R_2^2 - R_1^2)(\psi - \pi/4)} + o((\psi - \pi/4)^{-1}), \qquad \psi \to \frac{\pi}{4},$$
$$\xi_{r\theta} = -\frac{U_0 R_1}{(R_2^2 - R_1^2)(3\pi/4 - \psi)} + o((3\pi/4 - \psi)^{-1}), \qquad \psi \to \frac{3\pi}{4}. \tag{32}$$

With allowance for Eq. (14), we find from Eq. (13) that

$$\psi - \frac{\pi}{4} = \frac{\sqrt{2}\,(r - R_1)^{1/2}}{R_1^{1/2}(1 - R_1^2/R_2^2)^{1/2}} + o((r - R_1)^{1/2}), \qquad r \to R_1,$$
$$\frac{3\pi}{4} - \psi = \frac{\sqrt{2}\,(R_2 - r)^{1/2}}{R_2^{1/2}(R_2^2/R_1^2 - 1)^{1/2}} + o((R_2 - r)^{1/2}), \qquad r \to R_2. \tag{33}$$

It follows from Eqs. (32) that $|\xi_{r\theta}| \to \infty$ as $\psi \to \pi/4$ and $\psi \to 3\pi/4$. As the other components of the strain-rate tensor are bounded, we obtain the following relations from Eqs. (30), (32), and (33):

$$\xi_{\rm eq} = \frac{U_0(1 + R_1^2/R_2^2)}{\sqrt{6}\,R_1^{1/2}(1 - R_1^2/R_2^2)^{1/2}}\,(r - R_1)^{-1/2} + o((r - R_1)^{-1/2}), \qquad r \to R_1,$$
$$\xi_{\rm eq} = \frac{2U_0}{\sqrt{6}\,R_2^{1/2}(1 - R_1^2/R_2^2)^{1/2}}\,(R_2 - r)^{-1/2} + o((R_2 - r)^{-1/2}), \qquad r \to R_2. \tag{34}$$

In the case considered, the distance to the friction surface $r = R_1$ is determined by the equation $s = r - R_1$, and the distance to the friction surface $r = R_2$ is determined by the equation $s = R_2 - r$. Thus, the distribution of the equivalent strain rate near the friction surfaces, which is described by Eqs. (34), is consistent with the general theory [Eq. (29)], and the strain-rate intensity factors are determined from Eqs. (34):

$$D_1 = \frac{U_0(1 + R_1^2/R_2^2)}{\sqrt{6}\,R_1^{1/2}(1 - R_1^2/R_2^2)^{1/2}}, \qquad D_2 = \frac{2U_0}{\sqrt{6}\,R_2^{1/2}(1 - R_1^2/R_2^2)^{1/2}} \tag{35}$$

(the coefficients $D_1$ and $D_2$ correspond to the surfaces $r = R_1$ and $r = R_2$, respectively).

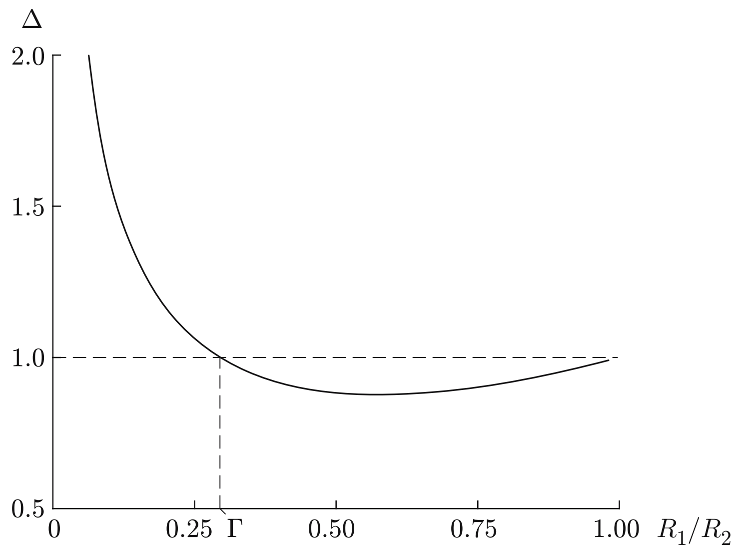


Fig. 4. Ratio of the strain-rate intensity factors $\Delta$ versus the ratio of the radii $R_1/R_2$.

As the knowledge of the strain-rate intensity factor allows us to predict the changes in material properties in a thin layer near the friction surface [3], but the relations between the value of this factor and the parameters characterizing material properties are not quantified, it seems of interest to analyze the ratio of the strain-rate intensity factors as a function of the ratio of the radii $R_1/R_2$ for the friction surfaces $r = R_1$ and $r = R_2$. From Eqs. (35), we find

$$\Delta = \frac{D_1}{D_2} = \frac{1 + R_1^2/R_2^2}{2(R_1/R_2)^{1/2}}. \tag{36}$$

The dependence of $\Delta$ on the ratio $R_1/R_2$ is plotted in Fig. 4. It is seen that $\Delta = 1$ at a certain value $R_1/R_2 = \Gamma$. Using Eq. (36) and performing simple transformations, we can reduce the equation $\Delta = 1$ to the third-order equation with respect to $R_1/R_2$. It follows from a simpler numerical solution that $\Gamma \approx 0.3$. It is seen in Fig. 4 that $\Delta > 1$ for $R_1/R_2 < \Gamma$ and $\Delta < 1$ for $R_1/R_2 > \Gamma$. Thus, in accordance with theoretical predictions [2, 3], the physical processes are more intense near the friction surface $r = R_1$ for $R_1/R_2 < \Gamma$ and near the friction surface $r = R_2$ for $R_1/R_2 > \Gamma$. Using this conclusion, it is possible to perform an experimental study to validate the general concept, which is the basis for the theory, without establishing exact quantitative dependences. In particular, the value of $\Delta^2$ equals the ratio of the characteristic lengths (thicknesses of the layers of high-intensity strains near the friction surfaces), which were introduced in [2].

It can be demonstrated that rotational motion of the friction surfaces in the case considered does not affect the strain-rate intensity factor. Indeed, let us assume that a cylinder of radius $R_1$ rotates with a velocity at which the shear stress at $r = R_1$ has the same sign as in Eq. (1). In this case, it is only condition (5) that is changed in the problem formulation, because the rigid zone should rotate together with the cylinder. This condition, however, affects only the quantity $u_0$ introduced in Eq. (27). As $u_0$ does not enter Eq. (35), rotational motion of the cylinder does not affect the strain-rate intensity factor. The reason is that the direction of the contact shear stresses induced by the rotational motion considered coincides with the direction of contact shear stresses arising under compression without rotation. Note that this condition is not usually satisfied in industrial processes [4–6], where the additional shear stress induced by instrument rotation is perpendicular to the stress arising in processes without rotation. For a plane strain state, it is impossible to obtain a model of such a distribution of shear stresses.

If the velocity of cylinder rotation in the resultant solution is assumed to be such that the shear stress at $r = R_1$ changes its sign to the opposite to that in Eq. (1), then the approximate solution of the class considered does not exist.

To conclude, we should note that the solution of the perfect plasticity theory equations in polar coordinates, from which the above-considered solution could be derived, was proposed in [15]. In that paper, however, the boundary-value problem necessary to obtain dependence (36) was not solved.

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